Non-Orthogonal Random Access with Channel Inversion for 5G Networks

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***Abstract*—In this paper, we propose a non-orthogonal random access (NORA) technique for the uplink 5G networks, in which user equipments (UEs) make use of the *channel inversion* so that their received power at the base station (BS) can satisfy a certain threshold condition. In particular, each UE *opportunistically* adjusts its transmit power according to its channel gain. We also mathematically analyze the system throughput and energy efficiency (EE) of the proposed NORA. Finally, we show the performance of the proposed NORA technique through extensive computer simulations.**

I. Introduction

To improve the spectral efficiency (SE) for the 5th generation (5G) mobile networks, non-orthogonal multiple access (NOMA) has been proposed [1], in which the receiver separates the super-imposed signals with the successive interference cancellation (SIC) technique. For the downlink, the base station (BS) constructs the multiplexed signal for a group of user equipments (UEs) in the same resource and allocates different transmission powers to each UE. At each UE in the downlink, the multiplexed signal in the downlink experiences the same fading and path-loss *collectively*, and it can be successfully separated by each UE with SIC technique if a proper power allocation is adopted at the BS. For the uplink, in contrast, the BS receives the super-imposed signals from different UEs, each of which experiences *independent* short-term (small-scale) fading and path-loss. Accordingly, the BS cannot successfully separate the multiple signals from UEs without each UE's proper power control according to both the fading and path-loss. For the uplink NOMA, the outage probability and sum-rate are investigated in [2], [3], in which the transmit power control (TPC) technique is utilized at each UEs in order to compensate the path-loss. However, only two or three UEs are considered.

Compared with [2], [3], this paper considers a random access network, where the population of UEs follows a Poisson process and the BS adopts the SIC technique, which is called *non-orthogonal random access (NORA)* system. In particular, we propose an opportunistic power control for UEs to employ according to their channel gain which includes not only the path-loss but also the small-scale fading, which is called channel inversion. As a main result, we mathematically analyze the system throughput (packets/slot), and the energy efficiency (EE) (packets/slot/joule) of the proposed NORA with channel inversion.

II. System model

Suppose a time division duplex (TDD) *uplink* wireless network, where there exists a BS at the center of a circular coverage area with radius R. We assume that UEs are randomly deployed in the area and communicate with the BS over the shared wireless link. Time is assumed to be divided into slots of a constant size, which is defined as time-duration for a packet transmission. We also assume that each UE can hold only a single packet and the aggregate packet transmissions, i.e., new and retransmissions by UEs are modeled as Poisson process with mean rate G (packets/slot). Before (re)transmitting a packet over the uplink, each UE is assumed to measure its channel gain $Y = hr^{-\alpha}$ through a reference signal of the downlink with channel reciprocity of TDD; h indicates a short-term fading, exponentially distributed random variable with unit mean; $r \in [0, R]$ is the distance from the BS and α denotes the path-loss exponent. If a UE finds $Y \geq \beta$, it sends a packet by adjusting its transmission power so that the received power at the BS is equal to P_1. If the UE finds $\beta_o \leq Y < \beta$, it sends its packet by adjusting its transmission power so that the received power at the BS is eqaul to P_2. We assume that $P_1 > P_2$. The UE does not (re)transmit its packet if $Y < \beta_o$, where β_o is called *outage threshold*. More precisely, let $P_{T,i}$ be the transmission power of the UE which targets at P_i. Then, $P_{T,i}$ for $i = 1, 2$ can be written as

$$P_{T,i} = \frac{P_i}{hr^{-\alpha}} = \frac{P_i}{Y}. \quad (1)$$

With the target received powers, P_1 and P_2, we assume that the BS employs the SIC-based receiver, which is a widely used receiver in power domain NOMA systems. In the SIC-based receiver, the BS always succeeds in decoding one packet with P_1, when it receives only one packet. Further, if the BS receives more than one packets, for the successful decoding of the packet with P_1, we can consider the following condition:

$$\frac{P_1}{kP_2 + N_0} \geq \gamma \quad \text{for} \quad k = 0, 1, \ldots, \quad (2)$$

where N_0 and γ denote noise power and SINR threshold for the successful decoding, respectively; k indicates the number of received packets with P_2. Let us define κ^* as the maximum number of the packets with P_2 so that the packet with P_1 is still successfully decoded. Using (2), we can get $\kappa^* \geq 1$ as

$$\kappa^* = \lfloor (P_1/\gamma - N_0)/P_2 \rfloor = \lfloor (\mathsf{SNR}_1 - \gamma)/(\gamma \mathsf{SNR}_2) \rfloor, \quad (3)$$

where $\mathsf{SNR}_i \triangleq P_i/N_0$ for $i=1$ and 2. Notice that if there exist more than one packets with P_1, no packets can be decoded successfully. For the packet with P_2 to be decoded successfully we have two cases: While there is only one packet with P_2, we should have either no packet, or only one with P_1. The first case is that no packet with P_1 is received at the BS, while only one packet with P_2 is received at the BS. In this case, if $\mathsf{SNR}_2 \geq \gamma$, then the packet is successfully decoded. The second case corresponds to the case when two packets received at the BS, each of which is received with P_1 and P_2, i.e., $k=1$ in (2). After the packet with P_1 is successfully decoded, it is removed with the SIC technique and the packet with P_2 is also successfully decoded if $\mathsf{SNR}_2 \geq \gamma$. Finally, we assume that the feedback from the BS is instantaneous so that UEs can know the channel outcome, i.e., success, before the next slot.

III. Performance Analysis

This section analyzes the system throughput and the EE, which are denoted by τ (packets/slot) and τ_E (packets/slot/joule), respectively.

Lemma 1: Let p be the probability that a UE targets at P_1 if its channel gain $Y \geq \beta$. For $\alpha = 4$, i.e., a typical value of pathloss exponents for urban area, it is expressed as

$$p = \Pr[Y \geq \beta | Y \geq \beta_o] = \sqrt{\frac{\beta_o}{\beta}} \frac{\mathrm{erf}\left(\sqrt{\beta}R^2\right)}{\mathrm{erf}\left(\sqrt{\beta_o}R^2\right)}, \quad (4)$$

where $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$.

Proof: Let $F_Y(y)$ be the cumulative distribution function (CDF) of Y, i.e., $F_Y(y) = \Pr[Y \leq y]$. It is written as

$$\begin{aligned} F_Y(y) &= \Pr\left[hr^{-\alpha} \leq y\right] = \int_0^R \left(1 - e^{-yr^\alpha}\right) f_R(r) dr \\ &= 1 - \int_0^R e^{-yr^\alpha} f_R(r) dr, \end{aligned} \quad (5)$$

where $f_R(r) = \frac{2r}{R^2}$ for $0 \leq r \leq R$. For $\alpha=4$, we have

$$F_Y(y) = 1 - \frac{2}{R^2} \int_0^R e^{-yr^4} r dr = 1 - \frac{1}{2R^2} \sqrt{\frac{\pi}{y}} \mathrm{erf}\left(\sqrt{y}R^2\right). \quad (6)$$

We can get (4) with $p = \frac{1-F_Y(\beta)}{1-F_Y(\beta_o)}$. ■

Using Lemma 1, the system throughput τ can be obtained as follows.

Proposition 1: Let G_1 and G_2 be the mean rate of Poisson processes (packets/slot), which is the packet arrival process for the target recieved power P_1 and P_2 at the BS. The system throughput is then obtained as

$$\tau = e^{-G}\left[G_2(1+G_1) + G_1 \sum_{k=0}^{\kappa^*} \frac{G_2^k}{k!}\right], \quad (7)$$

where we have $G_1 = pG$ and $G_2 = (1-p)G$, and κ^* is given in (3).

Proof: We denote by s_i for $i=1$ and 2 the random variable that a packet received with P_i is successfully decoded. Then, τ is expressed as

$$\begin{aligned} \tau = &\Pr[s_1=1, s_2=0] + \Pr[s_1=0, s_2=1] \\ &+ 2\Pr[s_1=1, s_2=1]. \end{aligned} \quad (8)$$

Before obtaining each probability on the right-hand side (RHS) in (8), let us remind that a UE targets at P_1 if it finds $Y \geq \beta$, given that $Y \geq \beta_o$. Since the aggregate packets (re)transmitted per slot obey the Poisson process with mean G, the packet arrivals with target P_1 and P_2 form Poisson process with mean rate $G_1 = pG$ and $G_2 = (1-p)G$, respectively. Returning to (8), we get the first term of its RHS, which is the probability that only one packet with P_1 is successfully decoded. This is expressed as

$$\Pr[s_1=1, s_2=0] = \sum_{k=0, k\neq 1}^{\kappa^*} G_1 e^{-G_1} \frac{G_2^k}{k!} e^{-G_2}. \quad (9)$$

We further get the second term on the RHS in (8) as

$$\Pr[s_1=0, s_2=1] = e^{-G_1} G_2 e^{-G_2}. \quad (10)$$

Finally, we have

$$\Pr[s_1=1, s_2=1] = G_1 e^{-G_1} G_2 e^{-G_2}. \quad (11)$$

Plugging (9)-(11) into (8) yields (7). ■

Proposition 2: When $P_1 = \theta P_2$ for $\theta > 1$, the EE of uplink NOMA with a two-level of target powers is

$$\tau_E = \frac{\tau}{GP_2(\theta\mathcal{Y}(\beta) + \overline{\mathcal{Y}}(\beta))/(1 - F_Y(\beta_o))}, \quad (12)$$

where $\mathcal{Y}(\beta)$ and $\overline{\mathcal{Y}}(\beta)$ indicate the expectation of the inverse of the channel gain for P_1 and P_2, respectively, which are given below.

Proof: We define EE as a ratio of the system throughput to the average transmission power consumption:

$$\tau_E \triangleq \frac{\tau}{(\mathsf{E}_\beta[P_{T,1}] + \mathsf{E}_\beta[P_{T,2}])G/(1 - F_Y(\beta_o))}, \quad (13)$$

where $\mathsf{E}_\beta[P_{T,i}]$ denote the average transmission power of a UE aiming at $P_{T,i}$ for $i=1,2$. We can write

$$\mathsf{E}_\beta[P_{T,1}] = P_1 \int_\beta^\infty \frac{1}{y} f_Y(y) dy = P_1 \mathcal{Y}(\beta), \quad (14)$$

in which $f_Y(y) = \frac{dF_Y(y)}{dy}$ and from (5) we have $f_Y(y) = \int_0^R r^\alpha e^{-yr^\alpha} f_R(r) dr$. For $\alpha = 4$, we can rewrite $\mathcal{Y}(\beta)$ in (14) as

$$\begin{aligned} \mathcal{Y}(\beta) &= \int_\beta^\infty \frac{1}{y} f_Y(y) dy \\ &= \frac{1}{R^2} \int_\beta^\infty \frac{1}{\sqrt{y^5}} \int_0^{\sqrt{y}R^2} z^2 e^{-z^2} dz dy \\ &= \int_\beta^\infty \left[\frac{\sqrt{\pi}}{4R^2\sqrt{y^5}} \mathrm{erf}\left(\sqrt{y}R^2\right) - \frac{1}{2y^2} e^{-R^4 y}\right] dy. \end{aligned} \quad (15)$$

The average transmission power of UEs for P_2 is expressed as

$$\mathsf{E}_\beta[P_{T,2}] = P_2 \int_{\beta_o}^\beta \frac{1}{y} f_Y(y) dy = P_2 \overline{\mathcal{Y}}(\beta). \quad (16)$$

Using (15), we readily get $\overline{\mathcal{Y}}(\beta) = \mathcal{Y}(\beta_o) - \mathcal{Y}(\beta)$. Assuming $P_1 = \theta P_2$ and using (14) and (16), we have (12). ■

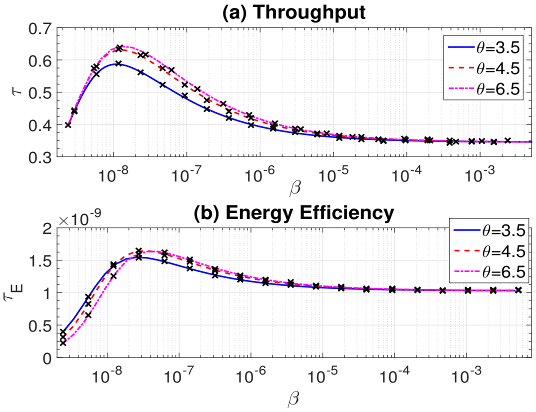

Fig. 1. The effect of β on the throughput τ and EE τ_E.

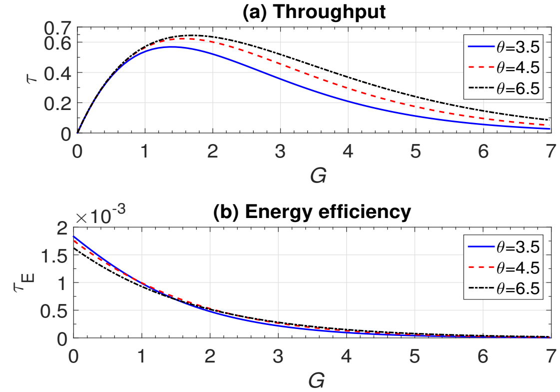

Fig. 2. The throughput τ and EE τ_E with G: $\gamma = 1.75$, $\beta = 5.537 \times 10^{-9}$, and $\beta_o = 1.9423 \times 10^{-9}$.

Let us discuss how P_1 and P_2 can be chosen. Given a bandwidth B, we can set P_2 as $\mathsf{SNR}_2 = P_2/N_0 = (2^{R_2/B} - 1) \geq \gamma$, where R_2 is the required data rate of UEs aiming at P_2. In choosing P_1, we set $P_1 = \theta P_2$ for $\theta > 1$. For $\mathsf{SNR}_2 = \gamma$, i.e., the minimum power P_2 to meet R_2, we write (3) as

$$\kappa^* = \left\lfloor \gamma^{-1} (\theta - 1) \right\rfloor \quad \text{for} \quad \theta > 1. \tag{17}$$

We can choose an optimal θ^*, equivalently optimal $P_1 = \theta P_2$, by maximizing (12).

IV. Numerical Results

We first discuss how to set β and β_o. The first method is to satisfy the probability that UEs do not experience the outage with constraint ϵ_0, i.e., $\Pr[hr^{-\alpha} \geq \beta_o] = \epsilon_0$. Using (6) we can find such a β_o numerically. For example, if $\epsilon_0 = 0.75$ and 0.9 for $R = 150$, we have $\beta_o = 1.9423 \times 10^{-9}$ and 6.5247×10^{-10}, respectively. This implies that if λ denotes the average density of UEs deployed per unit area, with $\epsilon_0 = 0.75$, it can be said that 75% of UEs on average, i.e., $G = \epsilon_0 \lambda \pi R^2$, are statistically able to communicate with the BS. Once β_o is determined, we then find β of maximizing τ_E. The second method is to restrict the average transmission power consumption of UEs. More specifically, we set $\mathcal{Y}(\beta) = \epsilon_1$ and $\overline{\mathcal{Y}}(\beta) = \epsilon_2$, which possibly corresponds to a total power consumption criterion, i.e., $P_1\epsilon_1 p + P_2\epsilon_2(1-p)$. Given ϵ_1 and ϵ_2, we can find β and β_o numerically, e.g., $\beta = \mathcal{Y}^{-1}(\beta)$. Note that, in setting γ, we assume $\frac{P_2}{N_0} = \gamma$ and $N_0 = 1$ from simplicity. Referring to (17), it can be seen that κ^* increases as $\gamma \to 0$, but the spectral efficiency for UE with P_2, i.e., $\log_2(1+\gamma)$, is expected to decrease.

To visualize the first method, Figs. 1 (a) and (b) show τ and τ_E by varying β for $G = 1.4$, $\gamma = 1.75$, $\beta_o = 1.9423 \times 10^{-9}$ and $\epsilon_0 = 0.75$, where symbol '$\times$' indicates simulation results. It is seen that the optimal β of maximizing τ and τ_E is 1.123×10^{-8} and 5.537×10^{-9}, respectively. It is interesting to see that in general the maximum of τ_E and τ is not achieved at the same β. As β increases, UEs shall target at P_2 only, so that the system is reduced to S-ALOHA systems. With a higher θ, i.e., a higher $E_\beta[T_1]$, τ is improved while τ_E is decreased. Note that the order of the magnitude of τ_E is extremely low, i.e., -9. This results from the low value of β_o used, which leads to some extremely high transmission power consumption for channel inversion upon the occurrences of very low channel gains. In order to have some desirable value of τ_E, we use the second method. If some transmission power constraint $\epsilon_1 = \mathcal{Y}(\beta)$ is given, one can choose β so that UEs targetting at P_1 consume the average transmission power $P_1\epsilon_1$. For example, for $\epsilon_1 = 10$ we have $\mathcal{Y}^{-1}(\epsilon_1) = \beta = 1.1995 \times 10^{-4}$, whereas $\beta_o = 1.5803 \times 10^{-5}$ is found for $\epsilon_2 = 200$. Note that in this case we have $p = 0.363$ in (4) and $\Pr[hr^{-\alpha} \geq \beta_o] = 0.01$. By reducing the transmission power consumption, the outage occurs more often, which incurs long access delay. Fig. 2 shows τ and τ_E with various G's and $\beta = 5.537 \times 10^{-9}$. Compared to Fig. 1, the order of the magnitude τ_E is -3. As G increases, τ_E gradually decreases due to the increase of collisions.

V. Conclusions

In this paper, we proposed a novel NORA system adopting the opportunistic channel inversion at UEs, and analyzed the throughput and EE of the proposed NORA system. We also discussed two ways of setting the thresholds of channel gain, at which UEs decide which target received power should be. It was observed that the throughput of NORA can exceed 0.6 by adjusting the channel gain thresholds and the transmission power consumption.

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